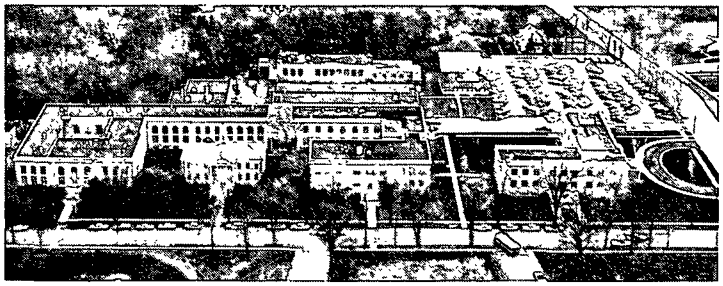

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ON THE EFFICIENCY OF BROWNSTOCK WASHING ON ROTARY FILTERS

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Abstract

For brownstock washing on rotary filters, the local, stage and overall efficiencies are defined. These form a hierarchy beginning with the most fundamental, the local efficiency, which is directly related to the rates of the underlying separation processes. Next is the stage efficiency which is most useful in characterizing the performance of a given filter stage. Finally, there is the overall efficiency which contains no detail about the process, but is simply a ratio of numbers of hypothetical to actual stages. The three efficiencies are related one to another without ambiguity. Single stage material balances are performed. These allow calculation of the stage efficiency in terms of the wash ratio, the recycle ratio and the local efficiency.

A detailed analysis of mass transfer and fluid displacement in the wash zone is performed. This yields a simple, direct relationship between the local efficiency and the number of transfer units. This equation is modified to account for possible air entrainment in the pad which can render a portion of the shower water ineffective with respect to displacement.

Performance equations are developed which give the wash loss directly in terms of wash ratio, stage efficiency and number of stages. Graphical depictions of calculation procedures are presented based on the concepts of operating and "Norden" lines. A minimum wash ratio is defined, and design equations yielding the required actual stages in terms of stage efficiency, wash loss and actual wash ratio are developed.

The results compare very favorably with actual mill data.

Introduction

Brownstock washing on rotary filters is a countercurrent staged operation. As such, this separation process is amenable to analysis by classical chemical engineering methods. These techniques have broad applicability to such common chemical engineering operations as distillation and extraction. In fact, the principles of analysis of these processes can be directly applied to brownstock washing by analogy.

At the most elementary level, a cascade of filters can be considered as a countercurrent array of black boxes. As such, simple material balance provides the locus of interstage concentrations, commonly referred to as the operating line for the system. This line is a convenient tool for tracking pulp and filtrate concentrations from one end of the cascade to the other.

At a more detailed level, one must distinguish between the liquor carried with the pulp and the wash water flowing through it. Although these are not distinct phases in the thermodynamic sense, they are physically distinguishable and can be compared by analogy, for example, to the vapor and liquid phases in distillation. Thus, there is a driving force for mass transfer between these "phases" because of the difference in solute concentrations. In addition, there is the potential for physical exchange, or displacement, between portions of these phases. This latter phenomenon does not occur in true two-phase separations, so any classical analysis of the underlying rate must be modified to account for it.

In washing, there also is no thermodynamic phase equilibrium relationship which would govern the limiting or "ideal" behavior of the cascade. There is, however, a convenient reference relationship in which the solute concentrations in the overflow and underflow from a stage are taken to be equal. This can be taken as playing the role of an "equilibrium" line, and a proper local efficiency defined relative to it. Once this is recognized, the entire formalism of nonequilibrium staged operations can be applied in a straightforward manner.

The Hierarchy of Efficiencies

The operation of a cascade of rotary filters can be depicted on a plot of solute concentration in the mobile wash water *versus* solute concentration in the liquor carried with the pulp. From Fig. 1, a material balance between the last (mth) filter and any other (nth) gives the operating line:

$$\overline{X}_n = \frac{1}{N} C_{n-1} + \overline{X}_{m+1} - \frac{1}{N} C_m \qquad (1)$$

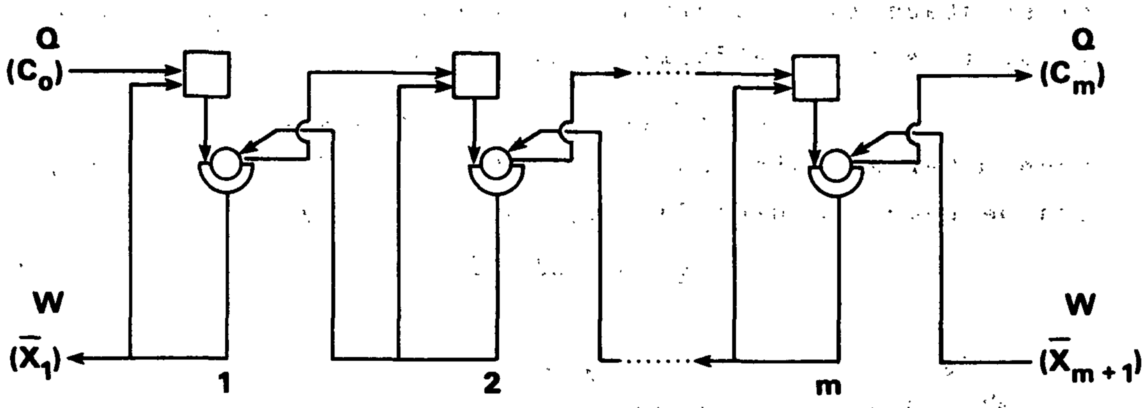

Fig. 1. Countercurrent cascade of filters.

The "equilibrium" line is defined as

$$\overline{X}_n = C_n \qquad (2)$$

This is a useful reference line and is appropriately called the Norden line.

Three different efficiencies, all interrelated, can be defined. First, with reference to Fig. 2, a local efficiency can be defined by

$$X_n - \overline{X}_{n+1} = E\ (C - \overline{X}_{n+1}) \qquad (3)$$

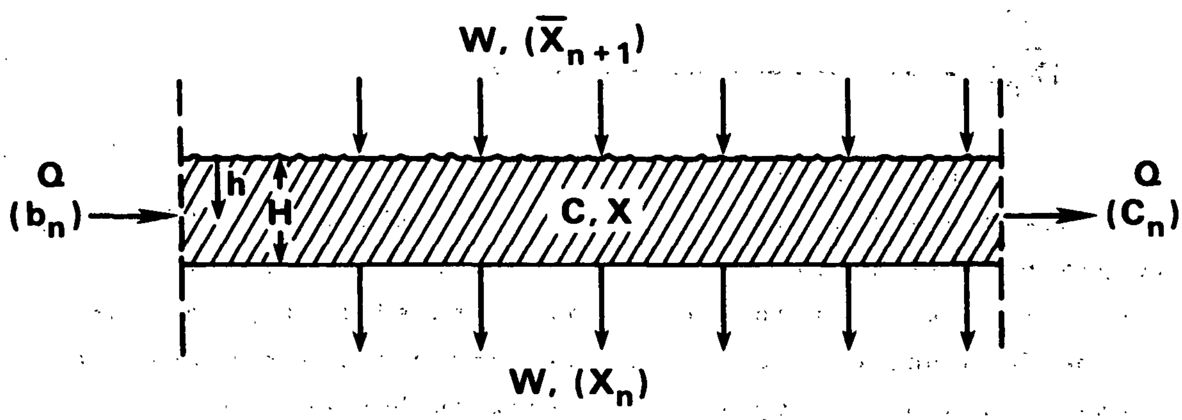

Fig. 2. Schematic of the wash zone.

Clearly, the value of E is determined by the rates of the underlying transport processes occurring in the wash zone. The dominant mechanisms are mass transfer and displacement, and in the next section a model is developed to predict the local efficiency by accounting for these phenomena.

The utility of the local efficiency lies in the fact that, once its value is determined either from first principles or from direct measurement, the performance of a given stage is determined in terms of operating parameters. Thus, if a stage efficiency is defined by

$$\overline{X}_n - \overline{X}_{n+1} = E_s\ (C_n - \overline{X}_{n+1}) \tag{4}$$

then material balances around the stage depicted in Fig. 3 can be combined with Eq. (3) to obtain (1):

$$E_s = \frac{(1+R)e^{EN} - 1}{R + N} \tag{5}$$

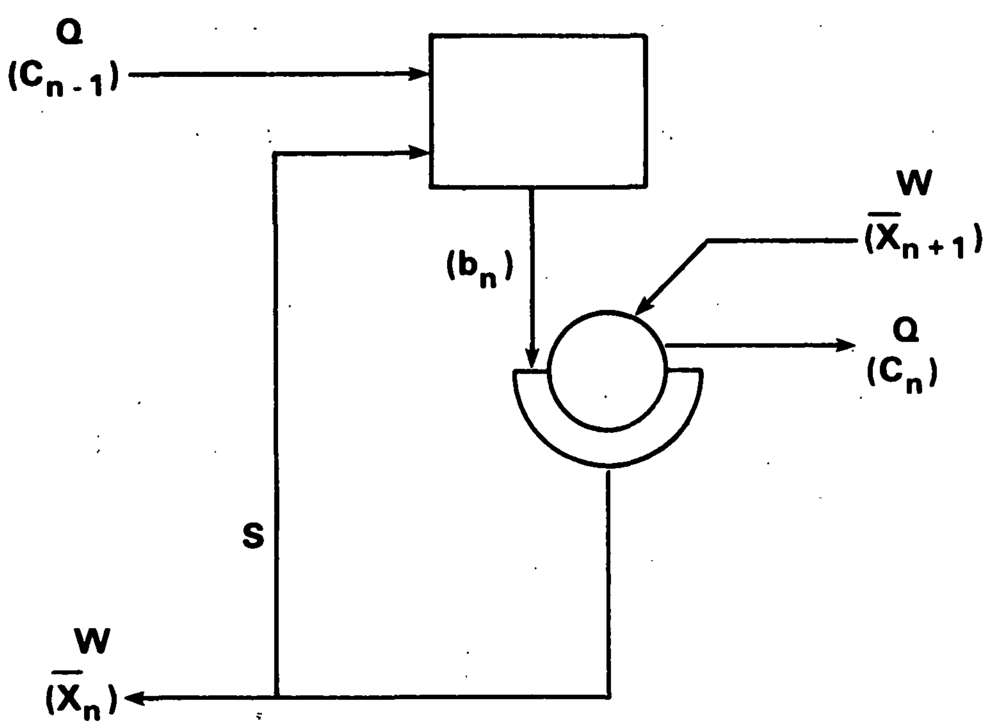

Fig. 3. The continuous rotary filter.

Once the local efficiency is known, the stage efficiency is given in terms of the wash and recycle ratios. The utility of the stage efficiency is illustrated in Fig. 4. Here the performance of the cascade is conveniently depicted in terms of a simple stepwise construction between the operating and equilibrium lines.

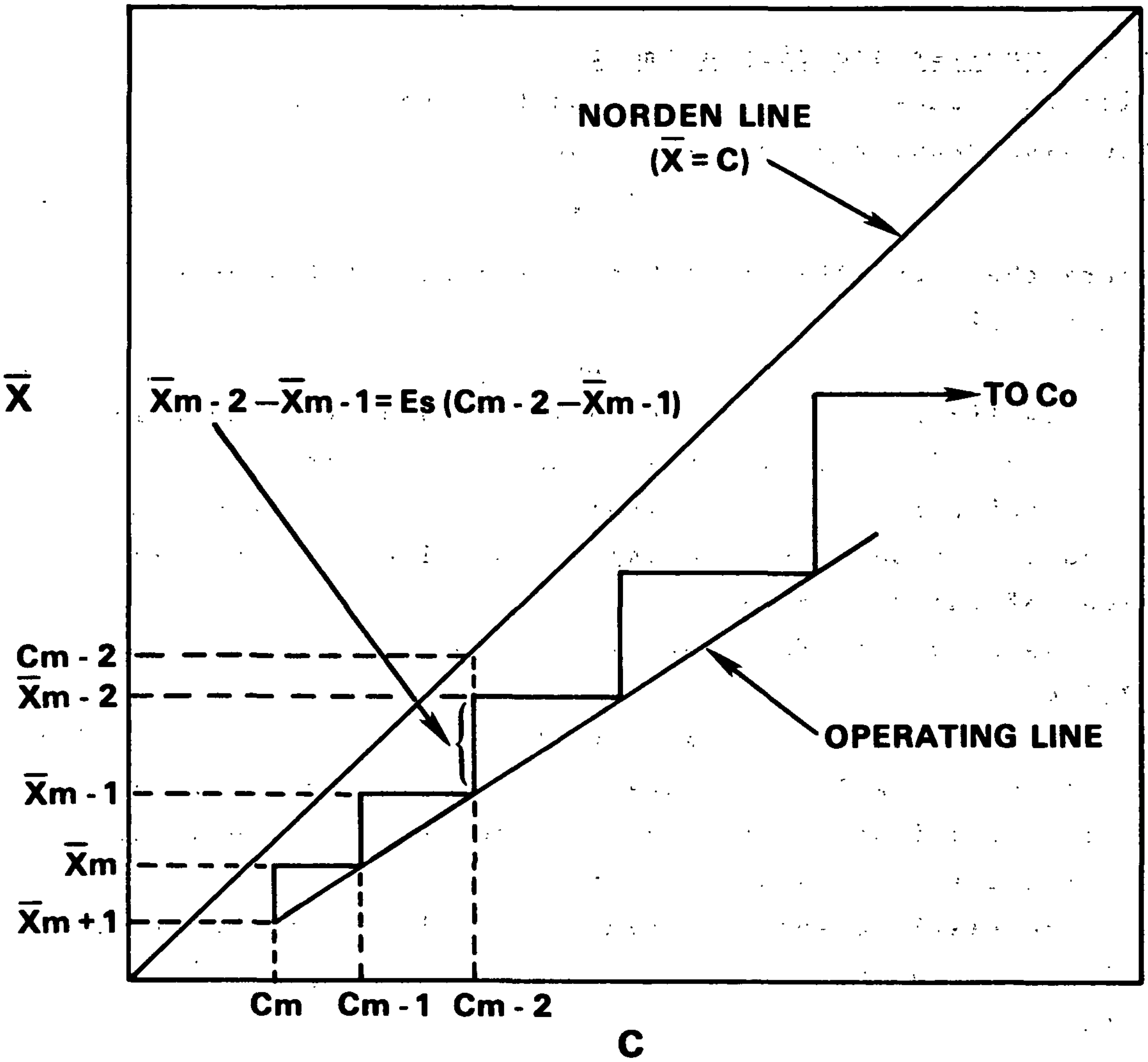

Fig. 4. Calculation for actual stages.

Another type of efficiency can be defined by noting the special case of Es = 1. In this hypothetical case, the stages are ideal in the Norden sense, and the graphical procedure involves simple steps between the operating and Norden lines. In this context, it is convenient to define an overall efficiency as

$$E_o = \frac{m_N}{m} \tag{6}$$

which directly relates the actual number of stages to the number of Norden stages required to achieve the same separation. It can be shown (1) that

$$E_o = \frac{\ln q}{\ln N} \tag{7}$$

where

$$q = 1 + Es\ (N - 1) \tag{8}$$

Therefore, E_o is a unique function of Es which, in turn, is a unique function of E.

Mass Transfer and Displacement

With reference to Fig. 2, a mass balance on the solute in the wash water can be written as

$$\hat{W} \frac{dX}{dh} = N_T a \tag{9}$$

where the total flux is taken to consist of two distinct contributions:

$$N_T = N_M + N_D \tag{10}$$

The first term, N_M, accounts for mass transfer under the influence of the difference in solute concentration between the flowing and stagnant liquors. The second term, N_D, accounts for physical exchange or displacement between portions of the two "phases."

The mass transfer flux can be simply expressed as (2,3,4)

$$N_M = k\ (C-X) \tag{11}$$

where the overall resistance to mass transfer (1/k) consists of contributions offered by both the stagnant liquor in the pad and the flowing wash water.

The displacement flux can be calculated from a simple balance on the portion of the pad occupied by flowing wash water:

$$V_w\ C = V_w\ \overline{X}_{n+1} + N_D a\ V_T \tau \tag{12}$$

where

$$\tau = \frac{H}{U_W} \tag{13}$$

and the interstitial velocity is related to the superficial velocity by

$$U_W\ V_W = \hat{W}\ V_T \tag{14}$$

Equations (12)-(14) can be combined to give

$$N_D a = \frac{\hat{W}}{H} (C - \overline{X}_{n+1}) \tag{15}$$

which can be termed the equivalent displacement flux (per unit volume).

Equations (10), (11), and (15) can then be substituted into the balance of Equation (9). The following form is obtained if the vertical variation of solute concentration in the pad is neglected.

$$\frac{d\ (C-X)}{dh} = - \frac{ka}{\hat{W}} (C-X) - \frac{1}{H} (C - \overline{X}_{n+1}) \tag{16}$$

This equation can be simply integrated and combined with Eq. (2) to obtain

$$E = (1 - e^{-\eta})\left(1 + \frac{1}{\eta}\right) \tag{17}$$

where

$$\eta = \frac{kaH}{\hat{W}} \tag{18}$$

which is the number of transfer units or the dimensionless mass transfer coefficient.

If air is entrained in the pad, a portion of the shower water can be rendered ineffective with respect to displacement. In this case, the analysis can be modified to yield the following:

$$E = (1 - e^{-\eta})\left(1 + \frac{1-\alpha}{\eta}\right) \tag{19}$$

where α is the fraction of the shower water rendered ineffective with respect to displacement because of the presence of air in the pad. Figure 5 is a plot of Eq. (19). Clearly aeration has a significant negative impact on the efficiency. With no aeration, the local efficiency is uniquely determined by η and lies in the range $1 < E < 1.3$. Our estimate of η for actual washers indicates that practical values fall in the range $1 < \eta < 3$. In this range, the local efficiency is close to its maximum value of 1.3.

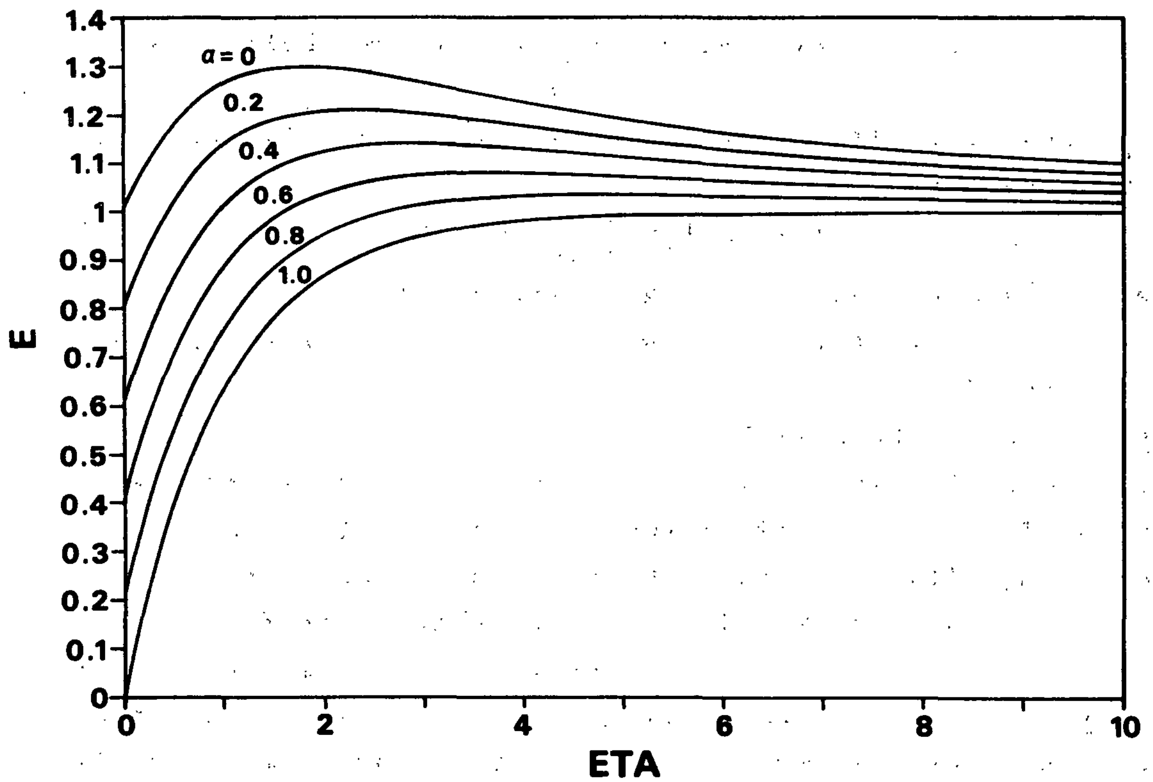

Fig. 5. The local efficiency.

Performance and Design Equations

With the local efficiency determined, the stage efficiency is set according to Eq. (5). The operation of the entire cascade is then fixed as illustrated in Fig. 4. Analytical equivalents of the graphical construction can be derived (1). For example:

$$\frac{C_m}{C_o} = L_m = 1 - \left[\frac{N(q^{m}-1)}{Nq^{m}-1}\right]\left(1 - \frac{\overline{X}_{m+1}}{C_o}\right) \tag{20}$$

where q is given by Eq. (8). This is a straightforward and useful performance relationship.

For design relationships, Eq. (20) can be inverted. In this context, it is useful to consider Fig. 4 in the Norden limit ($E_s = 1$). One can imagine the slope of the operating line ($^1/N$) increasing (at fixed $\overline{X}_m + 1$, C_m) until intersection of the operating and Norden lines occurs. The slope of this limiting operating line gives the minimum wash ratio, at which an infinite number of stages would be required for the specified loss ratio. The minimum wash ratio is given (1) as:

$$N_{min} = \frac{1 - L_m}{1 - \frac{\overline{X}_{m+1}}{C_o}} \tag{21}$$

A practical wash ratio must be some multiple of N or

$$N = \beta\, N_{min} \tag{22}$$

Equation (20) can be inverted using Eq. (21) and (22). The result is

$$m = \ell n \left[\frac{(\beta - 1)}{\beta\,(N_{min} - 1)}\right] / \ell n\ q \tag{23}$$

which is a convenient design relationship. Also, Eq. (23) can be used to calculated m_N in Eq. (6), by letting q = N [see Eq. (8)].

Comparison with Mill Data

The central feature of the present analysis is the prediction of the local efficiency given by Eq. (17) or its extension, Eq. (19). To provide an adequate test, data are required on washers which are essentially air-free. The data of Harper (5) for a three-stage washer and of Perkins *et al*. (6) for the first stage of a three-stage sequence are suitable for this purpose. The results are summarized in Table 1.

Table 1. Local Efficiency from Mill Data

Harper, 1985

Stage	E	η
1	1.25	1.8
2	1.09	1.2
3	1.35	1.5
Average	1.25	1.5

Perkins, 1954
(Stage 1)

N	E
1.0	1.34
1.22	1.24
1.44	1.19
1.88	1.13

The local efficiencies of these filters cluster close to the maximum value predicted by Eq. (17) which corresponds to the top curve in Fig. 5. This is consistent with our estimate that η lies in the range of $1 < \eta < 3$ for typical operating conditions. These results suggest that a local efficiency of 1.2-1.3 can be used with confidence in washer calculations provided aeration is avoided.

The pulp in the second and third stages of Perkins *et al*. (6) were reported to contain entrained air. The local efficiencies calculated for these stages were approximately 0.5 for the second stage and 0.7 for the third. This clearly demonstrates the significant detrimental impact of air entrainment.

Conclusion

The local efficiency, defined by Eq. (3) and predicted by Equation (17), is a very useful basis for washer calculations on cascades of rotary filters. The analysis presented here relates this fundamental quantity to overall performance and design relationships. The associated graphical constructions are also instructive aids to visualizing the process calculations. For completeness, we note that this new performance parameter, E, is related to the classic displacement ratio by the simple expression (1):

$$DR = 1 - e^{-EN} \qquad (24)$$

Finally, the analysis presented here can also be applied directly to other methods of brownstock washing on counter-current stages, such as belt filters or paper machine-type washers.

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Nomenclature

a	= effective mass transfer area per unit volume
b_n	= solute concentration in the reslurried feed to the nth filter
C	= solute concentration in the cake within the wash zone
C_n	= solute concentration in the cake exiting the nth filter
C_o	= solute concentration in the pulp entering the first stage of the cascade
DR	= displacement ratio
E	= local efficiency
E_s	= stage efficiency
E_o	= overall efficiency
H	= thickness of filter cake
k	= mass transfer coefficient
L_m	= overall system loss ratio
m	= total stages in the cascade
N	= the wash ratio = W/Q
N_{min}	= minimum wash ratio
n	= general stage index
q	= defined by Eq. (8)
Q	= volumetric rate of liquor held in filter cake
R	= the recycle ratio = S/Q
S	= volumetric rate of wash liquor recycle
U_w	= interstitial shower water velocity
V_w	= volume of pad occupied by flowing wash water
V_T	= total volume of pad
$\hat{W}$	= effective velocity of wash liquor in the wash zone
W	= volumetric wash flow rate
X	= solute concentration in the wash liquor within the wash zone
$\overline{X}_{n+1}$	= solute concentration in the wash liquor feed to the Nth stage
X_n	= solute concentration in the wash liquor leaving any point in the wash zone of the nth stage
α	= fractional bypass of shower water
β	= N/N_{min}